Transfer of Nonclassical Properties from a Microscopic Superposition to Macroscopic Thermal States in the High Temperature Limit

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We present several examples where prominent quantum properties are transferred from a microscopic superposition to thermal states at high temperatures. Our work is motivated by an analogy of Schrödinger's cat paradox, where the state corresponding to the virtual cat is a mixed thermal state with a large average photon number. Remarkably, quantum entanglement can be produced between thermal states with nearly the maximum Bell-inequality violation even when the temperatures of both modes approach infinity.

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Introduction.—It is a fundamental issue in quantum theory to understand how quantum properties such as quantum interference and entanglement can be transferred from microscopic objects to macroscopic classical systems. Schrödinger's cat paradox is probably the best known example of a counterintuitive situation arising from the interaction between quantum and classical worlds [1,2]. In Schrödinger's original paradox and its various explanations, the initial ''cat'' put into the box is considered to be in a pure state such as $|\text{alive}\rangle$. The macroscopic system, the cat, then interacts with a microscopic atomic system and becomes entangled with it. According to this argument a superposition of the macroscopic system such as $(|\text{alive}\rangle + |\text{dead}\rangle)/\sqrt{2}$ may be produced by measuring the microscopic part on a superposed basis.

However, in the original gedanken experiment, it is obvious that the macroscopic system has been interacting with the environment before being placed in the box. These interactions can cause the macroscopic system to become entangled with the environment before it interacts with the atomic system. Even though the box is ideally sealed, the macroscopic system will remain entangled with the environment due to its preinteractions with the environment. In such a case, the macroscopic system cannot be assumed as a pure state but it should be considered a mixed state [3]. Therefore, a more reasonable assumption would be that the macroscopic system was initially in a mixed state before it interacted with the microscopic superposition. These observations naturally lead to the question: if the cat in Schrödinger's paradox was in a mixed state, how would it be affected by the interaction with a microscopic superposition? Would this assumption wash out the quantum nature of the resulting state? This question clearly also relates more generally to the issue of the quantum to classical interface.

In this work, we consider such an interaction between a microscopic superposition and a significantly mixed thermal state at a high temperature. A thermal state with a very high temperature is considered a classical state in quantum optics. When the temperature approaches infinity, the thermal state does not show any quantum properties. As a comparison, coherent states with large amplitudes are known as the ''most classical'' pure states [4], and their superposition is sometimes regarded as a superposition of classical states [5]. However, coherent states are not strictly classical as they can be used for quantum key distribution [6] and display other nonclassical features [7].

Our examples in this Letter show that prominent quantum properties can be transferred from a microscopic superposition to significantly mixed thermal states at high temperatures through an experimentally feasible process. Remarkably, we find that quantum entanglement can be produced between thermal states with nearly the maximum Bell-inequality violation [i.e., up to Cirel'son's bound [8]] even when the temperatures of both modes go to infinity. To our knowledge, this interesting result has not been previously described. For example, Ferreira *et al.* recently showed that entanglement can be generated at any finite temperature between a high-Q cavity field and a movable mirror in a thermal state [9]. However, in their example [9] only one of the modes is considered a large thermal state and entanglement actually vanishes in the infinite temperature limit, which is obviously in contrast to our result. It is believed that high temperatures reduce entanglement and all entanglement vanishes if the temperature is high enough [9,10]. Our result overturns this belief and is distinguished from all the previous related works [9–12]. It also raises another interesting question on the possibility of efficient quantum information processing with high-temperature mixed systems.

Generating thermal-state superpositions.—Let us first consider a two-mode harmonic oscillator system. A displaced thermal state can be defined as $\rho^{\text{th}}(V, d) = \int d^2\alpha P^{\text{th}}(V, d)|\alpha\rangle\langle\alpha|$, where $|\alpha\rangle$ is a coherent state of amplitude α and $P^{\text{th}}(V, d) = \frac{2}{\pi(V-1)} \exp[-\frac{2|\alpha-d|^2}{V-1}]$ with V and D, the variance and the displacement in the phase space, respectively. The thermal temperature τ increases as V increases as $e^{\hbar\nu/\tau} = (V+1)/(V-1)$, where $\hbar$ is

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Planck's constant and ν is the frequency [13]. Suppose that a microscopic superposition state, $|\psi\rangle_a = (|0\rangle_a + |1\rangle_a)/2$, where $|0\rangle$ and $|1\rangle$ are the ground and first excited states of the harmonic oscillator, interacts with a thermal state $\rho_b^{\rm th}(V, d)$ and the interaction Hamiltonian is $H_\lambda = \lambda \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b}$ which corresponds to the cross-Kerr nonlinearity with the nonlinear strength λ. The resulting state is then

$$\rho_{ab}^{\rm ent} = \frac{1}{2}\int d^2\alpha P^{\rm th}(V, d)\{|0\rangle\langle 0| \otimes |\alpha\rangle\langle\alpha| + |1\rangle\langle 0| \otimes |\alpha e^{i\varphi}\rangle\langle\alpha| + |0\rangle\langle 1| \otimes |\alpha\rangle\langle\alpha e^{i\varphi}| + |1\rangle\langle 1| \otimes |\alpha e^{i\varphi}\rangle\langle\alpha e^{i\varphi}|\}, \quad (1)$$

where $\varphi = \lambda t$ and t is the interaction time. The Wigner representation of $\rho_{ab}^{\rm ent}$ is

$$W_{ab}^{\rm ent}(\alpha, \beta) = \frac{1}{\pi} e^{-2|\alpha|^2}\{W^{\rm th}(\beta; d) + 2\alpha V^c(\beta; d) + 2[\alpha V^c(\beta; d)]^* + (4|\alpha|^2 - 1)W^{\rm th}(\beta; de^{i\varphi})\}, \quad (2)$$

where α and β are complex numbers parametrizing the phase spaces of the microscopic and macroscopic systems, respectively, and

$$W^{\rm th}(\alpha; d) = \frac{2}{\pi V}\exp\left[-\frac{2|\alpha - d|^2}{V}\right], \quad (3)$$

$$V^c(\alpha; d) = \frac{2}{\pi JK}\exp\left[-\frac{2}{K}(1 - e^{i\varphi})d^2 - \frac{1}{J}\left(\alpha - \frac{2e^{i\varphi}d}{K}\right)\left(\alpha^* - \frac{2d}{K}\right)\right], \quad (4)$$

$K = 2 + (V - 1)(1 - e^{i\varphi})$, $J = (\sin\varphi/2 + iV\cos\varphi/2)/(2V\sin\varphi/2 + 2i\cos\varphi/2)$, and d has been assumed real without loss of generality. If one traces $\rho_{ab}^{\rm ent}$ over mode a, the remaining state will be simply in a classical mixture of two thermal states and its Wigner function will be nonnegative everywhere. However, if one measures out the "microscopic part" on the superposed basis, i.e., $(|0\rangle_a \pm |1\rangle_a)/\sqrt{2}$, the "macroscopic part" for mode b may not lose its nonclassical characteristics. Such a measurement on the superposed basis will reduce the remaining state to

$$\rho^{\rm sup(\pm)} = \mathcal{N}_s^\pm \int d^2\alpha P^{\rm th}(V, d)\{|\alpha\rangle\langle\alpha| \pm |\alpha e^{i\varphi}\rangle\langle\alpha| \pm |\alpha\rangle\langle\alpha e^{i\varphi}| + |\alpha e^{i\varphi}\rangle\langle\alpha e^{i\varphi}|\}, \quad (5)$$

where $\mathcal{N}_s^\pm$ are the normalization factors, and its Wigner function is $W^{\rm sup(\pm)}(\alpha) = \mathcal{N}_s^\pm\{W^{\rm th}(\alpha; d) \pm V^c(\alpha; d) \pm \{V^c(\alpha; d)\}^* + W^{\rm th}(\alpha; de^{i\varphi})\}$. The $\pm$ signs correspond to the two possible results from the measurement of the microscopic system. The states $\rho^{\rm sup(\pm)}$ can be understood as a generalization of the pure superpositions of coherent states [5,14] to high-temperature thermal mixtures.

Let us first suppose that $\varphi = \pi$. The state (5) then becomes $\rho^\pm \propto \rho^{\rm th}(V, d) \pm \sigma(V, d) \pm \sigma(V, -d) + \rho^{\rm th}(V, -d)$, where $\sigma(V, d) = \int d^2\alpha P^{\rm th}(V, d)|-\alpha\rangle\langle\alpha|$. If the initial state for mode b is a pure coherent state, i.e., $V = 1$, the measurement on the superposed basis for mode a will produce a superposition of two pure coherent states as $|\psi_\pm\rangle \propto |\alpha\rangle \pm |-\alpha\rangle$. The probability $\mathcal{P}_\pm$ to obtain the state $\rho^\pm$ is $\mathcal{P}_\pm = (1 \pm \exp[-2d^2/V])/2$. Suppose that both V and d are very large for the initial thermal state. Note that two thermal states $\rho^{\rm th}(V, \pm d)$ become macroscopically distinguishable when $d \gg \sqrt{V}$. We have found that both the states $\rho^\pm$ in this case show probability distributions with two Gaussian peaks and interference fringes. Figure 1 presents the probability distributions of $x(\equiv {\rm Re}[\alpha])$ and $p(\equiv {\rm Im}[\alpha])$ for ρ^- (a) when $V = 100$ and $d = 100$ and (b) when $V = 1000$ and $d = 300$. The probability distribution of x (p) for $\rho^\pm$ can be obtained by integrating the Wigner function of $\rho^\pm$ over p (x). The two Gaussian peaks along the x axis and interference fringes along the p axis shown in Fig. 1 are a typical signature of a quantum superposition between macroscopically distinguishable states. The visibility υ of the interference fringes is defined as [13] $\upsilon = (I_{\rm max} - I_{\rm min})/(I_{\rm max} + I_{\rm min})$, where $I = \int dx W^{\rm sup(-)}(\alpha)$ and the maximum and minimum should be taken over p. It can be simply shown that the visibility υ can be always 1 regardless of the value of V. Note that d should increase proportionally to $\sqrt{V}$ to maintain the condition of classical distinguishability between the two component thermal states $\rho^{\rm th}(V, \pm d)$. The interference fringes with high visibility are incompatible with classical physics and evidence of quantum coherence. The fringe spacing (the distance between the fringes) does not depend on V but only on d. A pure superposition of coherent states shows the same fringe spacing as the mixed

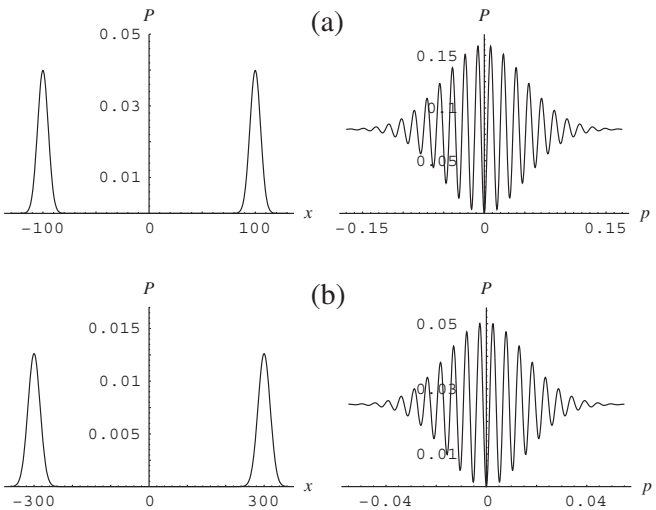

FIG. 1. The probability distributions P of x (left) and p (right) for a "superposition" of two distant thermal states. A thermal state with a large mixedness is converted to such a "thermal-state superposition" by interacting with a microscopic superposition. The variance V and displacement d for the thermal state are chosen as (a) $V = 100$ and $d = 100$, and (b) $V = 1000$ and $d = 300$. The fringe visibility is 1 regardless of V.

ones in Fig. 1 for a given d. We emphasize that the states shown in Fig. 1 are "superpositions" of severely mixed thermal states.

When d is small while V is still very large, i.e., a classical thermal state around the origin is assumed as the initial state, the resulting thermal-superposition states also show peculiar nonclassical behaviors. Figure 2 shows the probability distribution and the Wigner functions of the states (a) ρ^+ and (b) ρ^- where $V = 100$ and $d = 1$. As a result of the interaction with the microscopic superposition, a deep hole has been formed around the origin for ρ^- (or a sharp pole toward the top for the case of ρ^+). Interestingly, the Wigner function of ρ^- in Fig. 2(b) has a deep hole to the negative direction below zero. In this case, however, the probability $\mathcal{P}_-$ is only 1% while $\mathcal{P}_+$ is 99%.

An experimental realization of a nonlinear effect corresponding to $\varphi = \pi$ is very demanding particularly in the presence of decoherence. Here we point out that the method using a weak nonlinear effect ($\varphi \ll \pi$) combined with a strong field ($d \gg 1$) [15] can be useful to generate a thermal-state superposition with prominent interference patterns. In Fig. 3, we have used experimentally accessible values, $V = 5$, $d = 2000$, and $\varphi = \pi/1000$, but the fringe visibility is still 1. In this case, decoherence during the nonlinear interaction would be significantly reduced because of the decrease of the interaction time [15]. Note also that, if required, the state in Fig. 3 can be moved to the center of the phase space, for example, using a biased beam splitter (BS) and a strong coherent field [15].

Entanglement between thermal states.—There remains an important question concerning the possibility of generating entanglement between macroscopic objects and its Bell-type inequality tests [16]. We shall show that entanglement can be generated between high-temperature thermal states even when the temperature of each mode goes to infinity. If the microscopic superposition interacts with two thermal states, $\rho_b^{\rm th}(V, d)$ and $\rho_c^{\rm th}(V, d)$, and the microscopic particle is measured out on the superposed basis [17], the resulting state will be

$$\rho^{\rm tm(\pm)} \propto \rho^{\rm th}(V, d) \otimes \rho^{\rm th}(V, d) \pm \sigma(V, d) \otimes \sigma(V, d) \pm \sigma(V, -d) \otimes \sigma(V, -d) + \rho^{\rm th}(V, -d) \otimes \rho^{\rm th}(V, -d). \quad (6)$$

Thermal fields in two cavities and a two-level atom can be used to generate such a state. Here we shall consider the Bell-CHSH inequality [16,18] with photon number parity measurements [17,19]. Such parity measurements can be made in a high-Q cavity using a far-off-resonant interaction between a two-level atom and the field [20]. The Bell-CHSH inequality can be represented in terms of the Wigner function as [19]

$$|B^{(\pm)}| = \frac{\pi^2}{4}|W^{\rm tm(\pm)}(\alpha, \beta) + W^{\rm tm(\pm)}(\alpha, \beta') + W^{\rm tm(\pm)}(\alpha', \beta) - W^{\rm tm(\pm)}(\alpha', \beta')| \le 2, \quad (7)$$

where $W^{\rm tm(\pm)}(\alpha, \beta)$ is the Wigner function of $\rho^{\rm tm(\pm)}$ in Eq. (6). As shown in Fig. 4, the Bell-violation approaches the maximum bound for a bipartite measurement, $2\sqrt{2}$ [8], when $d \gg \sqrt{V}$ regardless of the level of the mixedness V, i.e., the temperatures of the thermal states. Note that it is true for both of $\rho^{\rm tm(+)}$ and $\rho^{\rm tm(-)}$ even though only the case of $\rho^{\rm tm(+)}$ has been plotted in Fig. 4(a). This implies that entanglement of nearly 1 ebit has been produced between the two significantly mixed thermal states for $d \gg \sqrt{V}$, and such "thermal-state entanglement" cannot be described by a local theory.

Slightly different types of macroscopic entanglement can be generated by applying the BS operation on the "thermal-state superpositions" in Eq. (5). When d is large, these states, which are generated by a 50:50 BS, violate the Bell-CHSH inequality to the maximum bound $2\sqrt{2}$ regardless of the level of mixedness V in exactly the same way described above. Furthermore, these states severely violate Bell's inequality even when $d = 0$ as V increases as shown in Fig. 4(b). We have found that the optimized Bell-violation of these states approaches 2.32449 for $V \to \infty$. Interestingly, this value is exactly the same as the opti-

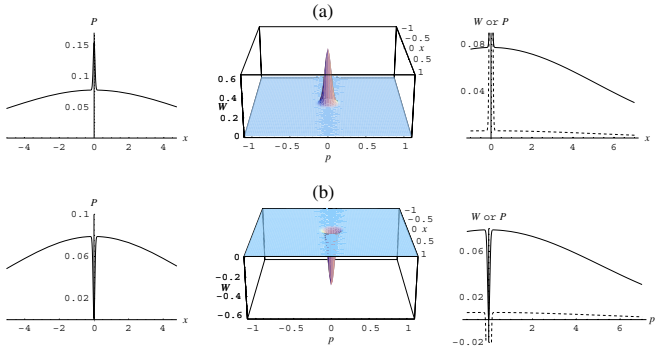

FIG. 2 (color online). The probability distributions P (left) and the Wigner functions W (middle) of the thermal state of $V = 100$ around the origin ($d = 1$) after an interaction with a microscopic superposition and a conditional measurement. The right-hand-side figures show that the background of the probability distributions (solid line) relates to the nonzero background of the Wigner functions (dashed line) around the sharp pole and the hole.

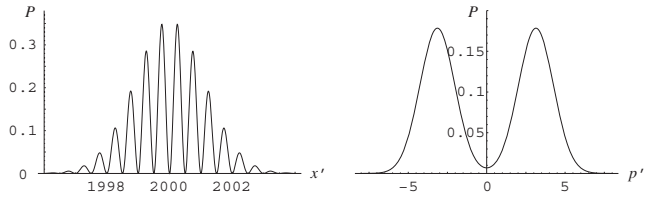

FIG. 3. The probability distributions P for a superposition of thermal states, $\rho^{\rm sup(-)}$, where $V = 5$, $d = 2000$, $\varphi = \pi/1000$. The x' (p') axis in this figure has been rotated by $\pi/2000$ from the x (p) axis for clarity.

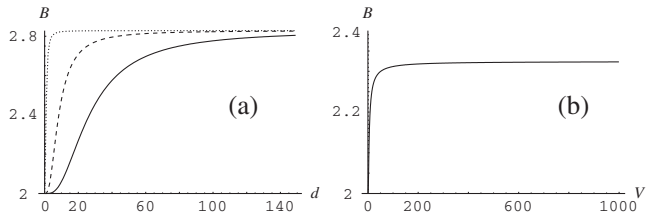

FIG. 4. (a) The optimized violation, $B \equiv |B^{(+)}|_{\max}$, of the Bell-CHSH inequality for the "thermal-state entanglement," $\rho^{\mathrm{tm}(+)}$, of $V = 1000$ (solid line) and $V = 100$ (dashed line). The Bell-violation of a pure entangled coherent state, i.e., $V = 1$, has been plotted for comparison (dotted line). (b) The optimized Bell violation B against V for the slightly different type of thermal-state entanglement generated using a 50:50 BS when $d = 0$. See text for details.

mized Bell-CHSH violation for a pure two-mode squeezed state in the infinite squeezing limit [21]. This is a remarkable distinguishing feature of the generation process of the thermal-state superpositions from that of the pure coherent-state superpositions using an initial coherent state [14]. If a coherent state with $d = 0$ (i.e., the vacuum) was used, the initial vacuum state would remain the same and no quantum effects described above can manifest.

Experimental feasibility.—A feasible experimental setup to test our examples is atom-field interactions in cavities where a $\pi/2$ pulse can be used to prepare the atom in a superposed state [14]. The two-mode thermal-state entanglement can be generated using two cavities and an atomic state detector [17]. Extending the two cavities to N, entanglement of N-mode thermal states can also be generated. Another possible setup is an all-optical scheme with free-traveling fields and a cross-Kerr medium, where a standard single-photon qubit could be used as the microscopic superposition [15]. Entanglement in a traveling field configuration can be produced at a BS. The observation of interference fringes can be performed using homodyne detection, which is generally very efficient, in a very short time.

Finally, we assess decoherence effects on quantum nonlocality (i.e., Bell-inequality violations) in a feasible range for photon number parity measurements. The decoherence analysis can be performed using the known solution of the master equation for a coherent-state dyadic: $|\alpha\rangle\langle\beta| \rightarrow e^{-(1-e^{-\gamma t})\{(|\alpha|^2+|\beta|^2)/2-\alpha\beta^*\}}|e^{-\gamma t/2}\alpha\rangle\langle e^{-\gamma t/2}\beta|$, where γ is the energy decay rate and t is time [15]. As can be expected, the increase of either V or d causes a more rapid destruction of nonlocality. However, when V takes moderate values, the condition for nonlocality ($|B^{(\pm)}|_{\max} > 2$) can persist fairly long compared to typically discussed pure cases. We suggest an experimentally feasible case, i.e., $V = 3$ and $d = 1$. In this case, the degree of mixedness of the initial thermal state in terms of the linear entropy ($M = 1 - \mathrm{Tr}[\rho^2]$) is ≈ 0.67, i.e., a very mixed state. However, nonlocality of a generated state, ρ^-, divided at a 50:50 BS persists slightly longer (until $\gamma t \approx 0.13$) than a pure superposition state, $|\alpha\rangle + |-\alpha\rangle$, with $\alpha = 2.2$ divided at a 50:50 BS (until $\gamma t \approx 0.12$). Here, the total average photon number for ρ^- (at $t = 0$) is $\approx$2.5, i.e., the average photon number detected at each detector is only $\approx$1.3. Then, there is a good possibility of performing parity measurements using current technology. As another case, if $V = 10$ (then $M = 0.9$) and $d = 0$, the survival time of nonlocality for the generated state ρ^+ divided at a 50:50 BS is approximately the same as that of a pure coherent-state superposition with $\alpha = 3.55$ (until $\gamma t \approx 0.05$). The average photon number for each detector in this case is $\approx$2.0. In summary, we have shown that superpositions of "classical" thermal states still show strong quantum effects, even when temperatures approach infinity, and that their small scale demonstrations at fairly low temperatures appear experimentally feasible.

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